UDK 517

NECESSARY CONDITIONS BELONGING TO MIXED TYPE EQUATIONS IN THE STRIP AND THE SOLUTION OF THIS PROBLEM

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Abstract: Border problem in the strip for the first composition, the mixed type two-dimensional differential equation with special derivative with fixed coefficient was considered and the obtained necessary conditions were studied in the work. In both part fundamental solution is in the direction of x_2. Here it was determined the value of Heaviside function depending on the complex argument. The solution of the problem was determined by the first parts of the obtained main relations.

Key words and phrases: First composition mixed type equation, border problem in strip, fundamental solution, main relations, necessary conditions, studying of necessary conditions.

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НЕОБХОДИМЫЕ УСЛОВИЯ, ПРИНАДЛЕЖАЩИЕ УРАВНЕНИЯМ СМЕШАННОГО ТИПА В ПОЛОСЕ, И РЕШЕНИЕ ЭТОЙ ЗАДАЧИ

Аннотация: В работе рассмотрена краевая задача в полосе для первого порядка, двумерного дифференциального уравнения смешанного типа со специальной производной с фиксированным коэффициентом и изучены полученные необходимые условия. В обеих частях основное решение находится в направлении. Здесь определялось значение функции Хевисайда в зависимости от комплексного аргумента. Решение задачи определялось первыми частями полученных основных соотношений.

Ключевые слова: уравнение смешанного типа первого порядка, краевая задача в полосе, фундаментальное решение, основные соотношения, необходимые условия, изучение необходимых условий.

INTRODUCTION

It is known that mixed type equation has been begun to be studied by Trikomi [1], then Gellersted [2], [3], Bitsadze [4] and his students were engaged in such equations [5], [6]. The border problems for the mixed type equations we studied are within the border conditions having local and global (integrals) limits [7]-[11].

The main relations (or relations) are built my beans of fundamental solution of the equation (sometimes the studied equation) attached to the equation given in this work. The second parts of these relations consisting of two parts are the necessary conditions. These conditions are studied by being separated.

THE FORMULATION OF PROBLEM

Review the following border problem leaning to real axis, whose width is unit, located in the upper semi-plane

$$\frac{\partial u_1(x)}{\partial x_2}+\frac{\partial u_1(x)}{\partial x_1}=0, x_1<0, x_2\in(0,1), \tag{1}$$

$$\frac{\partial u_2(x)}{\partial x_2}+i\frac{\partial u_2(x)}{\partial x_1}=0, x_1>0, x_2\in(0,1), \tag{2}$$

$$u_1(x_1,1)+\alpha_1 u_1(x_1,0)=\varphi_1(x_1), x_1\leqslant 0, \tag{3}$$

$$u_2(x_1,1)+\alpha_2 u_2(x_1,0)=\varphi_2(x_1), x_1\geqslant 0, \tag{4}$$

$$u_1(0,x_2)=u_2(0,x_2), x_2\in[0,1], \tag{5}$$

$$u_1(-\infty,x_2)+\alpha_0 u_2(\infty,x_2)=\varphi_0(x_2), x_2\in[0,1], \tag{6}$$

Here $i=\sqrt{-1}$, $D_1=\{x=(x_1,x_2): x_1<0, x_2\in(0,1)\}$, $D_2=\{x=(x_1,x_2): x_1>0, x_2\in(0,1)\}$, α_k, $k=\overline{0,2}$ — are given fixed figures, $\varphi_1(x_1)$, $\varphi_2(x_1)$ and $\varphi_0(x_1)$ given uninterrupted functions (correspondingly if $x_1\leqslant 0, x_1\geqslant 0$ and $x_2\in[0,1]$).

If we accept the following mark

$$u(x)=\begin{cases} u_1(x), & x_1<0, \quad x_2\in(0,1), \\ u_2(x) & x_1>0, \quad x_2\in(0,1). \end{cases} \tag{7}$$

Then equations (1), (2) will be as the following in case of $x\in D=D_1\cup D_2$:

$$\frac{\partial u(x)}{\partial x_2}+a(x_1)\frac{\partial u(x)}{\partial x_1}=0, x_1\in R, x_2\in(0,1), \tag{8}$$

here

$$a(x_1)=\frac{1+i}{2}(1+i\,sign x_1)=\begin{cases} 1, & x_1<0, \\ i, & x_1>0. \end{cases} \tag{9}$$

It is easy to see that, the fundamental solutions of equations (1) and (2) in the direction of $x_2$are as following functions:

$$U_1(x-\xi)=\theta(x_2-\xi_2)\,\delta(x_1-\xi_1-(x_2-\xi_2)), x_1,\xi_1<0; x_2,\xi_2\in(0,1), \tag{10}$$

$$U_2(x-\xi)=\theta(x_2-\xi_2)\,\delta(x_1-\xi_1-i(x_2-\xi_2)), x_1,\xi_1>0; x_2,\xi_2\in(0,1). \tag{11}$$

Thus, $\theta(t)$ — unified function of Heaviside, $\delta(t)$ — delta function of Dyrak.

MAIN RELATIONS

By multiplying given equation (1) with the fundamental solution (10), integrate for D_1 area. By applying Ostragradskiy–Gauss formula we get the following upon the second Green formula:

$$0=\int_{-\infty}^{0}dx_1\int_{0}^{1}\frac{\partial u_1(x)}{\partial x_2}U_1(x-\xi)\,dx_2+\int_{0}^{1}dx_2\int_{-\infty}^{0}\frac{\partial u_1(x)}{\partial x_1}U_1(x-\xi)dx_1=$$

$$=\int_{-\infty}^{0}dx_1\left[u_1(x)\,U_1(x-\xi)\big|_{x_2=0}^{1}-\int_{0}^{1}u_1(x)\frac{\partial U_1(x-\xi)}{\partial x_2}dx_2\right]+$$

$$+\int_0^1 dx_2 \left[u_1(x)\, U_1(x-\xi)\big|_{x_1=-\infty}^{0} - \int_{-\infty}^{0} u_1(x) \frac{\partial U_1(x-\xi)}{\partial x_1} dx_1 \right],$$

or

$$\int_{-\infty}^{0} u_1(x_1{,}1) U_1(x_1-\xi_1{,}1-\xi_2)\, dx_1 - \int_{-\infty}^{0} u_1(x_1{,}0) U_1(x_1-\xi_1, -\xi_2)\, dx_1 +$$

$$+\int_0^1 u_1(0{,}x_2) U_1(-\xi_1{,}x_2-\xi_2)\, dx_2 - \int_0^1 u_1(-\infty{,}x_2) U_1(-\infty-\xi_1{,}x_2-\xi_2)\, dx_2 =$$

$$= \int_{D_1} u_1(x) \left[\frac{\partial U_1(x-\xi)}{\partial x_2} + \frac{\partial U_1(x-\xi)}{\partial x_1} \right] dx = \int_{D_1} u_1(x) \delta(x-\xi)\, dx =$$

$$= \begin{cases} u_1(\xi), & \xi \in D_1 \\ \frac{1}{2} u_1(\xi) & \xi \in \partial D_1. \end{cases} \quad (12)$$

We get the followings by means of equation (2) and fundamental solution (11) in the same manner:

$$0 = \int_0^\infty dx_1 \int_0^1 \frac{\partial u_2(x)}{\partial x_2} U_2(x-\xi)\, dx_2 + i \int_0^1 dx_2 \int_0^\infty \frac{\partial u_2(x)}{\partial x_1} U_2(x-\xi) dx_1 =$$

$$= \int_0^\infty dx_1 \left[u_2(x)\, U_2(x-\xi)\big|_{x_2=0}^{1} - \int_0^1 u_2(x) \frac{\partial U_2(x-\xi)}{\partial x_2} dx_2 \right] +$$

$$+i \int_0^1 dx_2 \left[u_2(x)\, U_2(x-\xi)\big|_{x_1=0}^{\infty} - \int_0^\infty u_2(x) \frac{\partial U_2(x-\xi)}{\partial x_1} dx_1 \right],$$

or

$$\int_0^\infty u_2(x_1{,}1) U_2(x_1-\xi_1{,}1-\xi_2)\, dx_1 - \int_0^\infty u_2(x_1{,}0) U_2(x_1-\xi_1, -\xi_2)\, dx_1 +$$

$$+i \int_0^1 u_2(\infty{,}x_2) U_2(\infty-\xi_1{,}x_2-\xi_2)\, dx_2 - i \int_0^1 u_2(0{,}x_2) U_2(-\xi_1{,}x_2-\xi_2)\, dx_2 =$$

$$= \int_{D_2} u_2(x) \left[\frac{\partial U_2(x-\xi)}{\partial x_2} + i \frac{\partial U_2(x-\xi)}{\partial x_1} \right] dx = \int_{D_2} u_2(x) \delta(x-\xi)\, dx =$$

$$= \begin{cases} u_2(\xi), & \xi \in D_2, \\ \frac{1}{2} u_2(\xi) & \xi \in \partial D_2. \end{cases} \quad (13)$$

Each of obtained main relations (12) and (13) consists of two parts. The first parts give the analytical expression for any solution, the second parts the necessary conditions for equations (1) and (2) correspondingly. Separate them.

Necessary conditions:

$$\frac{1}{2} u_1(\xi_1{,}0) = \int_{-\infty}^{0} u_1(x_1{,}1) \theta(1)\, \delta(x_1-\xi_1-1)\, dx_1 - \int_{-\infty}^{0} u_1(x_1{,}0)\, \theta(0)\, \delta(x_1-\xi_1)\, dx_1 +$$

$$+\int_0^1 u_1(0,x_2)\,\theta(x_2-0)\,\delta(-\xi_1-x_2)\,dx_2-\int_0^1 u_1(-\infty,x_2)\theta(x_2-0)\,\delta(-\infty-\xi_1-x_2)\,dx_2=$$

$$=u_1(\xi_1+1,1)-\frac{1}{2}u_1(\xi_1,0)+u_1(0,-\xi_1),\xi_1<0,$$

or

$$u_1(\xi_1,0)=u_1(\xi_1+1,1)+u_1(0,-\xi_1),\xi_1<0, \tag{14}$$

$$\frac{1}{2}u_1(\xi_1,1)=\frac{1}{2}\int_{-\infty}^0 u_1(x_1,1)\delta(x_1-\xi_1)\,dx_1-\int_{-\infty}^0 u_1(x_1,0)\,\theta(-1)\,\delta(x_1-\xi_1+1)\,dx_1+$$

$$+\int_0^1 u_1(0,x_2)\,\theta(x_2-1)\,\delta(-\xi_1-x_2+1)\,dx_2-\int_0^1 u_1(-\infty,x_2)\theta(x_2-1)\,\delta(-\infty-\xi_1-x_2+1)\,dx_2,$$

as in the common differential equations, this condition became identity.

$$\frac{1}{2}u_1(0,\xi_2)=\int_{-\infty}^0 u_1(x_1,1)\theta(1-\xi_2)\,\delta(x_1-1+\xi_2)\,dx_1-\int_{-\infty}^0 u_1(x_1,0)\,\theta(-\xi_2)\,\delta(x_1+\xi_2)\,dx_1+$$

$$+\int_0^1 u_1(0,x_2)\,\theta(x_2-\xi_2)\,\delta(x_2-\xi_2)\,dx_2-\int_0^1 u_1(-\infty,x_2)\theta(x_2-\xi_2)\,\delta(-\infty-x_2+\xi_2)\,dx_2=$$

$$=u_1(1-\xi_2,1)+\frac{1}{2}u_1(0,\xi_2)-\theta(-\infty)\,u_1(-\infty,\xi_2-\infty)\ \ \xi_1<0,$$

or

$$u_1(1-\xi_2,1)=0,\xi_2\in(0,1), \tag{15}$$

$$\frac{1}{2}u_1(-\infty,\xi_2)=\int_{-\infty}^0 u_1(x_1,1)\theta(1-\xi_2)\,\delta(x_1+\infty-1+\xi_2)\,dx_1-$$

$$-\int_{-\infty}^0 u_1(x_1,0)\,\theta(-\xi_2)\,\delta(x_1+\infty+\xi_2)\,dx_1+$$

$$+\int_0^1 u_1(0,x_2)\,\theta(x_2-\xi_2)\,\delta(\infty-x_2+\xi_2)\,dx_2-\int_0^1 u_1(-\infty,x_2)\theta(x_2-\xi_2)\,\delta(-\infty+\infty-x_2+\xi_2)\,dx_2=$$

$$=u_1(1-\xi_2-\infty,1)+\theta(\infty)\,u_1(0,\xi_2+\infty)-\frac{1}{2}u_1(-\infty,\xi_2)$$

or

$$u_1(-\infty,\xi_2)=u_1(1-\xi_2-\infty,1)+u_1(0,\xi_2+\infty). \tag{16}$$

In the same manner we get the following necessary conditions from the main relation (13)

$$\frac{1}{2}u_2(\xi_1,0)=\int_0^\infty u_2(x_1,1)\delta(x_1-\xi_1-i)\,dx_1-\frac{1}{2}\int_0^\infty u_2(x_1,0)\,\delta(x_1-\xi_1)\,dx_1+$$

$$+i\int_0^1 u_2(\infty,x_2)\,\delta(\infty,-\xi_1-ix_2)\,dx_2-i\int_0^1 u_2(0,x_2)\delta(-\xi_1-ix_2)\,dx_2=$$

$$=u_2(\xi_1+i,1)-\frac{1}{2}u_2(\xi_1,0)-u_2(\infty,-i(\infty-\xi_1))+u_2(0,i\xi_1),$$

or

$$u_2(\xi_1,0)=u_2(\xi_1+i,1)-u_2(\infty,-i(\infty-\xi_1))+u_2(0,i\xi_1),\tag{17}$$

$$\frac{1}{2}u_2(\xi_1,1)=\frac{1}{2}\int_0^\infty u_2(x_1,1)\delta(x_1-\xi_1)\,dx_1-\int_0^\infty u_2(x_1,0)\,\theta(-1)\,\delta(x_1-\xi_1+i)\,dx_1+$$

$$+i\int_0^1 u_2(\infty,x_2)\,\theta(x_2-1)\,\delta(\infty-\xi_1-ix_2+i)\,dx_2-i\int_0^1 u_2(0,x_2)\theta(x_2-1)\,\delta(x_1-\xi_1-ix_2+i)\,dx_2,$$

this necessary condition also became identity as in the common differential equations

$$\frac{1}{2}u_2(0,\xi_2)=\int_0^\infty u_2(x_1,1)\delta(x_1-i+i\xi_2)\,dx_1-\int_0^\infty u_2(x_1,0)\,\theta(-\xi_2)\,\delta(x_1+i\xi_2)\,dx_1+$$

$$+i\int_0^1 u_2(\infty,x_2)\,\theta(x_2-\xi_2)\,\delta(\infty-ix_2+i\xi_2)\,dx_2-i\int_0^1 u_2(0,x_2)\theta(x_2-\xi_2)\,\delta(-i(x_2-\xi_2))\,dx_2=$$

$$=u_2(i-i\xi_2,1)-\theta(-i\infty)\,u_2(\infty,\xi_2-i\infty)+\frac{1}{2}u_2(0,\xi_2),$$

or

$$u_2(i-i\xi_2,1)-\theta(-i\infty)\,u_2(\infty,\xi_2-i\infty)=0.\tag{18}$$

Finally

$$\frac{1}{2}u_2(\infty,\xi_2)=\int_0^\infty u_2(x_1,1)\delta(x_1-\infty-i+i\xi_2)\,dx_1-\int_0^\infty u_2(x_1,0)\,\theta(-\xi_2)\,\delta(x_1-\infty+i\xi_2)\,dx_1+$$

$$+i\int_0^1 u_2(\infty,x_2)\,\theta(x_2-\xi_2)\,\delta(-i(x_2-\xi_2))\,dx_2-i\int_0^1 u_2(0,x_2)\theta(x_2-\xi_2)\,\delta(-\infty-ix_2+i\xi_2)\,dx_2=$$

$$=u_2(\infty+i-i\xi_2,1)-\frac{1}{2}u_2(\infty,\xi_2)+\theta(i\infty)\,u_2(0,\xi_2+i\infty)$$

or

$$u_2(\infty,\xi_2)=u_2(\infty+i-i\xi_2,1)+\theta(i\infty)\,u_2(0,\xi_2+i\infty).\tag{19}$$

Theorem 1. Any solution of equation (1)–(2) given in strip meet the necessary conditions (14)–(19).

Studying of necessary conditions:

Separate into two parts obtained condition (14):

$$\xi_1\in(-1,0)\,,u_1(\xi_1,0)=u_1(\xi_1+1,1)+u_1(0,-\xi_1),\tag{14$_1$}$$

$$\xi_1\in(-\infty,-1)\,,u_1(\xi_1,0)=u_1(\xi_1+1,1).\tag{14$_2$}$$

Obtained necessary condition (15) is identity, that's function $u_1(t{,}1)$ is not designated in $t \in (0{,}1)$, it is zero in the identity. (none) condition (16) is as following

$$u_1(-\infty{,}\xi_2) = u_1(-\infty{,}1), \xi_2 \in (0{,}1). \tag{16_1}$$

Condition (17) is related to the continuation of $u_2(x)$ to complex plane

$$u_2(\xi_1{,}0) = u_2(\xi_1 + i{,}1) - u_2(\infty{,}0 - i(\infty - \xi_1)) + u_2(0{,}0 + i\xi_1), \xi_1 \in (0{,}\infty). \tag{17_1}$$

In the same manner conditions (18) and (19) are maintained.

Note 1. As function $x_2 + ix_1$ is the solution of (2), it is clear from (18) and (19) that it should be

$$\theta(i\infty) = 0, \theta(-i\infty) = 1.$$

Thus, we get the following judgment.

Theorem 2. Any solution of given equation (1), (2) shall meet the following conditions:

$$u_1(\xi_1{,}0) = u_1(0, -\xi_1), \xi_1 \in (-1{,}0), \tag{20}$$

$$u_1(\xi_1{,}0) = u_1(\xi_1 + 1{,}1), \xi_1 \in (-\infty, -1), \tag{21}$$

$$u_1(-\infty{,}\xi_2) = u_1(-\infty{,}1), \xi_2 \in (0{,}1), \tag{22}$$

$$u_2(\xi_1{,}0) = u_2(\xi_1 + i{,}1) - u_2(\infty{,}i(\xi_1 - \infty)) + u_2(0{,}i\xi_1), \xi_1 \in (0{,}\infty), \tag{23}$$

$$u_2(i(1 - \xi_2){,}1) = u_2(\infty{,}\xi_2 - i\infty), \xi_2 \in (0{,}1), \tag{24}$$

$$u_2(\infty{,}\xi_2) = u_2(\infty + i(1 - \xi_2){,}1), \xi_2 \in (0{,}1). \tag{25}$$

Note 2. From the above theorem

$$u_2(\xi_1{,}0) = u_2(0{,}i\xi_1), \xi_1 \in (0{,}1) \tag{26}$$

the last two expressions are linear dependant and $u_1(-\infty{,}\xi_2), u_2(\infty{,}\xi_2)$ — fixed figures.

Now we get the following fundamental equation in order to determine $u_1(x_1{,}1)$ by using condition (3) and necessary condition (21)

$$u_1(x_1{,}1) = \varphi_1(x_1) - \alpha_1 u_1(x_1 + 1{,}1), x_1 \in (-\infty, -1). \tag{27}$$

If we get the following substitution from above:

$$u_1(x_1 + 1{,}1) = \frac{\varphi_1(x_1) - u_1(x_1{,}1)}{\alpha_1},$$

$$x_1 + 1 = t.$$

We get the following functional equation:

$$u_1(t{,}1) = \frac{\varphi_1(t-1) - u_1(t-1{,}1)}{\alpha_1}, \quad t \in (-\infty{,}0), \tag{28}$$

solve this problem by the method of writing in its place consecutively. As

$$u_1(t-1{,}1) = \frac{\varphi_1(t-2) - u_1(t-2{,}1)}{\alpha_1},$$

$$u_1(t{,}1) = \frac{\varphi_1(t-1) - \frac{\varphi_1(t-2) - u_1(t-2{,}1)}{\alpha_1}}{\alpha_1} = \frac{\varphi_1(t-1)}{\alpha_1} - \frac{\varphi_1(t-2)}{\alpha_1^2} + \frac{u_1(t-2{,}1)}{\alpha_1^2}, \quad t \in (-\infty{,}0). \tag{29}$$

If we continue this process, as

$$u_1(t-2,1) = \frac{\varphi_1(t-3)}{\alpha_1} - \frac{\varphi_1(t-4)}{\alpha_1^2} + \frac{u_1(t-4,1)}{\alpha_1^2},$$

from (29)

$$u_1(t,1) = \frac{\varphi_1(t-1)}{\alpha_1} - \frac{\varphi_1(t-2)}{\alpha_1^2} + \frac{\varphi_1(t-3)}{\alpha_1^3} - \frac{\varphi_1(t-4)}{\alpha_1^4} + \frac{u_1(t-4,1)}{\alpha_1^4}, \quad t \in (-\infty,0), \tag{30}$$

Finally

$$|\alpha_1| > 1, \tag{31}$$

if

$$u_1(t,1) = -\sum_{k=1}^{\infty} \frac{\varphi_1(t-k)}{\alpha_1^k}(-1)^k, t \in (-\infty,0), \tag{32}$$

As $u_1(-\infty,x_2)$ is limited, (it is clear from (6))

$$u_1(t,0) = \frac{\varphi_1(t) - u_1(t,1)}{\alpha_1}, t \in (-\infty,0). \tag{33}$$

If we accept $\xi_2 = i\xi_1$in (24) by considering the following obtained expression in (23)

$$u_2(\infty,i(\xi_1-\infty)) = u_2(i+\xi_1,1).$$

We get expression (26)

$$u_2(\xi_1,0) = u_2(0,i\xi_1), \quad \xi_1 \in (0,1),$$

$$u_2(0,x_2) = u_1(0,x_2) = u_1(-x_2,0) = \frac{\varphi_1(-x_2) - u_1(-x_2,1)}{\alpha_1}, \quad x_2 \in (0,1), \tag{34}$$

$u_1(-\infty,x_2)$ is determined by means of (22) and (32).

If we take into consideration (26), we get the following expression from (17)

$$u_2(\xi_1+i,1) = u_2(\infty,-i(\infty-\xi_1)). \tag{35}$$

It is obtained identity from (24) and (35).

Finally, by considering the followings from conditions (4) and (23)

$$u_2(x_1,1) + \alpha_2(u_2(x_1+i,1) - u_2(\infty,i(x_1-\infty)) + u_2(0,ix_1)) = \varphi_2(x_1), x_1 \geqslant 0,$$

or

$$u_2(x_1,1) + \alpha_2 u_2(0,ix_1) = \varphi_2(x_1), x_1 \geqslant 0, \tag{36}$$

and here (34), we get the following expression:

$$u_2(x_1,1) = \varphi_2(x_1) - \alpha_2\frac{\varphi_1(-ix_1) - u_1(-ix_1,1)}{\alpha_1}. \tag{37}$$

Finally $u_2(x_1,0)$ is determined from (4). The fixed $u_2(\infty,\xi_2)$ is determined from (25).

Qeyd 3: By using found border values, the solution of formulated border issue is gotten from main relations (12) and (13).

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