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SOLUTION OF CAUCHY AND BOUNDARY VALUE PROBLEMS FOR A DISCRETE POWERATIVE DERIVATIVE CUBIC EQUATION

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Abstract: As is known, the problems for the differential equations with continuously changing order of the derivatives are not considered completely.

It should be noted that this area is one of the less studied fields of modern mathematics and there are not effective methods for the study of problems for such differential equations, just as we study the problem for partial differential equations, with both with additive and a multiplicative derivatives.

In the present article, we will study the solution of Cauchy and boundary value problems for a discrete powerative derivative cubic equation. Using the definition given for this derivative, the cubic derivative in the equation is reduced, general solution is constructed analytically, which depends on three arbitrary constant equations. Then Cauchy and boundary value problems for this equation are considered. Arbitrary constants included in the general solution are determined from the given conditions, and an analytical expression is obtained to solve the problems.

Key words and phrases: discrete powerative derivative, discrete powerative cubic derivative, Cauchy problem, boundary value problem, general solution of the equation, analytical expression for the solution of the problem.

РЕШЕНИЕ КОШИ И КРАЕВЫХ ЗАДАЧ ДЛЯ ДИСКРЕТНОГО ПОВЕРАТИВНОГО ПРОИЗВОДНОГО УРАВНЕНИЯ ТРЕТЬЕГО ПОРЯДКА

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Аннотация: Как известно, задачи для дифференциальных уравнений с непрерывно меняющимся порядком производных полностью не исследованы.

Следует отметить, что эта область является одной из наименее изученных областей современной математики, и не существует эффективных методов исследования задач для таких дифференциальных уравнений, так как мы изучаем задачу для уравнений с частными производными, как с аддитивной, так и с мультипликативными производными.

В представленной статье будет исследовано решение Коши и краевых задач для дискретного поверативного производного уравнения третьего порядка. Используя определение, данное для этой производной, производная третьего порядка в уравнении уменьшается, аналитически строится общее решение, которое зависит от трех произвольных постоянных уравнения. Затем рассматриваются Коши и краевые задачи для этого уравнения. Произвольные постоянные, включенные в общее решение, определяются из заданных условий, для решения поставленных задач получается аналитическое выражение.

Ключевые слова: дискретная поверативная производная, дискретная поверативная производная третьего порядка, задача Коши, краевая задача, общее решение уравнения, аналитическое выражение решения задачи.

INTRODUCTION

It is known that additive derivatives of the equation and, both the Cauchy problem, and the boundary value problems for them were widely studied [1]–[5]. Despite the fact that discrete additive derivative equations, the so-called "differential equations", and problems for them [6]–[8] have been deeply studied, such equations are mainly formed by discretization of the problems for ordinary or special derivative equations [9]–[13]. Although the creation of a numerical chain and the Fibonacci sequence [finding a common limit] [14]–[15] leads to questions about direct discrete additive derivative equations, this direction has not been thoroughly investigated.

Although the multiplicative derivative was given half a century ago [16], the problems for these equations were recently considered [17].

Although the establishment of a general limit of the geometric sequence was set for a discrete multiplicative derivative equation, these questions were not well studied.

Although the powerative derivative (without pruning) is not yet known in the literature (for this, seven algebraic actions are not well known to us), we give here the definition of a discrete powerative derivative (for this, known algebraic actions are sufficient) and consider Cauchy and boundary value problems for such equations.

For such equations, we have considered some problems [18]–[19].

Note that if the transfer of each derivative (without pruning) requires two inverse actions, and the transfer of each discrete derivative requires one inverse action [20].

By the same rule, the transfer of each integral (without pruning) requires two direct actions, and the transfer of each discrete integral requires one direct action.

A PROBLEM STATEMENT: It is known that the discrete additive derivative

$$y_n^{(I)} = y_{n+1} - y_n, \tag{1}$$

discrete multiplicative derivative [18]

$$y_n^{[I]} = \frac{y_{n+1}}{y_n}, \tag{2}$$

finally the same discrete powerative derivative [19]–[20],

$$y_n^{\{I\}} = \sqrt[y_n]{y_{n+1}}, \tag{3}$$

thus determined.

Let's look at the equation as follows:

$$y_n^{\{III\}} \equiv \left(\left(y_n^{\{I\}}\right)^{\{I\}}\right)^{\{I\}} = f_n, n \geqslant 0, \tag{4}$$

here f_n, $n \geqslant 0$ is the given sequence, and y_n is the sequence under investigation.

If this equation (4) we write in open form:

$$y_{n+3}^{y_{n+2}^{-(1+y_{n+1}^{-1}+y_{n+1}^{-(1+y_n^{-1})})}} = f_n, n \geqslant 0. \tag{5}$$

We can get an equation with very complex non-linear differentiations. Here we consider the Cauchy and boundary value problems for equation (5), analyze and for solving these problems we obtain an analytical expression. We note that the Cauchy problem is considered for equation (5), that is, if y_0, y_1 and y_2 is given, then the existence of a solution to the Cauchy problem and the uniqueness of this solution are easily seen using the expression y_{n+3}, definable (5). But it is not so easy to show the analytical expression of the solution to this problem. As for the boundary value problem, it is impossible to say the idea of its solution.

Therefore, we return to equation (4) and mark it in the following form:

$$\left(y_n^{\{II\}}\right)^{\{I\}} = f_n, n \geqslant 0.$$

Here using the definition of a discrete verativ derivative:

$$\sqrt[y_n^{\{II\}}]{y_{n+1}^{\{II\}}} = f_n, n \geqslant 0,$$

or

$$y_{n+1}^{\{II\}} = f_n^{y_n^{\{II\}}}, n \geqslant 0. \tag{6}$$

Here we give n estimates and we get:

If $n = 0$, then

$$y_1^{\{II\}} = f_0^{y_0^{\{II\}}}, \tag{7}$$

expression, if $n = 1$

$$y_2^{\{II\}} = f_1^{y_1^{\{II\}}},$$

we get expression. If in the resulting expression we consider (7), this expression will be in the following form:

$$y_2^{\{II\}} = f_1^{f_0^{y_0^{\{II\}}}}. \tag{8}$$

If we continue this process, we will get from (6):

$$y_2^{\{II\}} = f_{n-1}^{f_{n-2}^{\cdot^{\cdot^{\cdot^{f_1^{f_0^{y_0^{\{II\}}}}}}}}}, n \geqslant 1. \tag{9}$$

We take the notation as follows:

$$g_n(y_0^{\{II\}}) \equiv f_{n-1}^{f_{n-2}^{\cdot^{\cdot^{\cdot^{f_1^{f_0^{y_0^{\{II\}}}}}}}}}, n \geqslant 1, \tag{10}$$

then (9) will be in the following form:

$$y_n^{\{II\}} = g_n(y_0^{\{II\}}), n \geqslant 1. \tag{11}$$

Thus, comparing (4) with (11), it appears that the cubic equation (4) reduces to the quadratic equation (11).

Therefore, equation (11)

$$\left(y_n^{\{I\}}\right)^{\{I\}} = g_n(y_0^{\{II\}}), n \geqslant 1,$$

writing in such a form, using the definition of a discrete powerative derivative, we will get:

$$\sqrt[y_n^{\{I\}}]{y_{n+1}^{\{I\}}} = g_n(y_0^{\{II\}}), n \geqslant 1,$$

or same

$$y_{n+1}^{\{I\}} = g_n^{y_n^{\{I\}}}, n \geqslant 1. \qquad (12)$$

Giving ratings n, we will get:

$$y_2^{\{I\}} = g_1^{y_1^{\{I\}}},$$

$$y_3^{\{I\}} = g_2^{y_2^{\{I\}}} = g_2^{g_1^{y_1^{\{I\}}}}.$$

Continuing this process:

$$y_n^{\{I\}} = g_{n-1}^{g_{n-1}^{\cdot^{\cdot^{\cdot^{g_2^{g_1^{y_1^{\{I\}}}}}}}}}, n \geqslant 2. \qquad (13)$$

And now, by analogy with (10), we adopt the following notation:

$$h_n(y_0^{\{II\}}, y_1^{\{I\}}) \equiv g_{n-1}^{g_{n-1}^{\cdot^{\cdot^{\cdot^{g_2^{g_1^{y_1^{\{I\}}}}}}}}}, n \geqslant 2. \qquad (14)$$

Then (13) will be as follows:

$$y_n^{\{I\}} = h_n(y_0^{\{II\}}, y_1^{\{I\}}), n \geqslant 2. \qquad (15)$$

Thus, after two stages, the cubic equation (4) is reduced to a quadratic equation (15). Finally, if we use the definition of the discrete powerative derivative in equation (15), we will get:

$$\sqrt[y_n]{y_{n+1}} = h_n, n \geqslant 2, \qquad (16)$$

or

$$y_{n+1} = {h_n}^{y_n}, n \geqslant 2. \qquad (17)$$

Here giving estimates n:

$$y_3 = {h_2}^{y_2},$$

$$y_4 = {h_3}^{y_3} = {h_3}^{{h_2}^{y_2}}.$$

Continuing this process:

$$y_n = {h_{n-1}}^{{h_{n-1}}^{\cdot^{\cdot^{\cdot^{{h_3}^{{h_2}^{{y_2}^{\{I\}}}}}}}}}. \qquad (18)$$

Thus, the general solution of equation (4) is obtained in the form of (18), where here $h_k(y_0^{\{II\}}, y_1^{\{I\}})$ with $g_s(y_0^{\{II\}})$ are determined by (14), and $g_s(y_0^{\{II\}})$ with f_m are defined by (10). Included in this common solution y_2, $y_1^{\{I\}}$ and $y_0^{\{II\}}$ is an arbitrary constant.

Thus, a general solution, which depends on three arbitrary constants for the cubic equation (4), is assigned.

The result will be stated in the following judgment.

Theorem 1. If f_n, $n \geqslant 0$ is a given sequence of positive elements, then the general solution of equation (4) can be represented as (18), here $h_k(y_0^{\{II\}}, y_1^{\{I\}})$, $g_s(y_0^{\{II\}})$ are determined by f_m with (10).

Now we have to define three arbitrary constants, such as y_2, $y_1^{\{I\}}$ and $y_0^{\{II\}}$, which are included in the general solution of equation (4) using the conditions (initial or boundary conditions) given in the problems we are considering.

Cauchy problem: Suppose that for equation (4)

$$y_k = \alpha_k, k = \overline{0,2}. \tag{19}$$

The initial conditions are given.

As we said above, using the data (19), we can define arbitrary constants that participate in the general solution:

$$y_2 = \alpha_2, \tag{20}$$

$$y_1^{\{I\}} = \sqrt[y_1]{y_2} = \sqrt[\alpha_1]{\alpha_2}, \tag{21}$$

$$y_0^{\{II\}} = \left(y_0^{\{I\}}\right)^{\{I\}} = \sqrt[y_0^{\{I\}}]{y_1^{\{I\}}} = \sqrt[y_0^{\{I\}}\sqrt{y_1\{I\}}]{\sqrt[y_1^{\{I\}}]{y_2^{\{I\}}}} = \sqrt[y_1^{y_0^{-1}}]{y_2^{y_1^{-1}}} = \left(y_2^{y_1^{-1}}\right)^{y_1^{y_0^{-1}}} =$$
$$= y_2^{y_1^{-1-y_0^{-1}}} = y_2^{y_1^{-(1+y_0^{-1})}} = \alpha_2^{\alpha_1^{-(1+\alpha_0^{-1})}}. \tag{22}$$

Thus, we get the following judgment.

Theorem 2. According to the conditions of Theorem 1, if α_k, $k = \overline{0,2}$ — are positive real numbers, then there is only one solution to the Cauchy problem (4), (19) and this solution is in (10), (14) and (18) should be taken into account in accordance with (20)–(22).

Boundary value problem: Suppose for equation (4)

$$y_0^{\{II\}} = \beta_0, y_1^{\{I\}} = \beta_1, y_N = \beta_2, \tag{23}$$

given the boundary conditions. Then expressions (10)

$$g_n(y_0^{\{II\}}) \equiv f_{n-1}^{f_{n-2}^{\cdot^{\cdot^{\cdot^{f_1^{f_0^{\beta_0}}}}}}}, \tag{24}$$

but (14)

$$h_n(y_0^{\{II\}}, y_1^{\{I\}}) = g_{n-1}^{g_{n-2}^{\cdot^{\cdot^{\cdot^{g_2^{g_1^{\beta_1}}}}}}}, \tag{25}$$

expression can be obtained. Then (18) (4), (23) to solve the boundary value problem:

$$y_n = h_{n-1}^{h_{n-2}^{\cdot^{\cdot^{\cdot^{h_3^{h_2^{y_2}}}}}}}, \tag{26}$$

we can get this expression.

If we use (23) the third of the boundary conditions, writing $n = N$ in expression (26), which we obtained:

$$\beta_2 = y_N = h_{N-1}^{h_{N-2}^{\cdot^{\cdot^{\cdot^{h_3^{h_2^{y_2}}}}}}}, \tag{27}$$

we can get the equation. From here we define y_2.

For this, we logarithm (27):

$$h_{N-2}^{h_{N-3}^{\cdot^{\cdot^{\cdot^{h_3^{h_2^{y_2}}}}}}} = \log_{h_{N-1}} \beta_2,$$

if we logarithm again:

$$h_{N-3}^{h_{N-4}^{\cdot^{\cdot^{\cdot^{h_3^{h_2^{y_2}}}}}}} = \log_{h_{N-2}} \log_{h_{N-1}} \beta_2,$$

if we continue this process:

$$y_2 = \log_{h_2} \log_{h_3} \ldots \log_{h_{N-2}} \log_{h_{N-1}} \beta_2, \tag{28}$$

we can get this expression.

Finally (23) and (28) we write in (18), then we will get:

$$y_n = \log_{h_n} \log_{h_{n+1}} \ldots \log_{h_{N-2}} \log_{h_{N-1}} \beta_2. \tag{29}$$

Thus, (4), (23) for solving the boundary value problem in the form (29), we can obtain an analytical expression.

Here $h_k(\beta_0, \beta_1)$ with (25), but $g_s(\beta_0)$ with (24) are determined.

Theorem 3. According to the conditions of Theorem 1, if $\beta_k > 0$, $k = \overline{0, 2}$, then there is only one solution to the boundary value problem (4), (23) and this solution in the form (29). Here $h_k(\beta_0, \beta_1)$ with (25), and $g_s(\beta_0)$ with (24) are defined.

Unsolved problem: if instead of condition (23)

$$y_0 = \gamma_0, y_1 = \gamma_1, y_N = \gamma_2, \tag{30}$$

if the boundary conditions are given, (4), (30) the problems how can be investigated.

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