

APPROXIMATE PROPERTIES OF THE DISCRETE PARTIAL FOURIER-JACOBI SUMS OF THE RECTANGULAR TYPE

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It is shown if $P_m^{\alpha,\beta}(x)$ ($\alpha, \beta > -1, m=0,1,2,\dots$) is the classical Jacobi polynomials, then the system of polynomials of two variables $\{\Psi_{mn}^{\alpha,\beta}(x,y)\}_{m,n=0}^r = \{P_m^{\alpha,\beta}(x)P_n^{\alpha,\beta}(y)\}_{m,n=0}^r$ ($r = m+n \leq N-1$) is an orthogonal system on the grid $\Omega_{N \times N} = \{(x_i, y_j)\}_{i,j=0}^N$, where x_i, y_j are the zeros of the Jacobi polynomial $P_N^{\alpha,\beta}(x)$. Given an arbitrary continuous function $f(x,y)$ on the square $[-1,1]^2$, if we construct the discrete partial Fourier–Jacobi sums of the rectangular type $S_{m,n,N}^{\alpha,\beta}(f;x,y)$ over the orthonormal system introduced above. We prove that the order of Lebesgue constant $\|S_{m,n,N}^{\alpha,\beta}\|$ of the discrete sums $S_{m,n,N}^{\alpha,\beta}(f;x,y)$ for $-1/2 < \alpha, \beta < 1/2, m+n \leq N-1$ is $O((mn)^{q+1/2})$, where $q = \max\{\alpha, \beta\}$. As a consequence of this result, we consider a several approximate properties of the discrete sums $S_{m,n,N}^{\alpha,\beta}(f;x,y)$.

The key words: Jacobi polynomial, Lebesgue function, Lebesgue constant, discrete set, best approximation, discrete partial Fourier–Jacobi sums, Christoffel's numbers.