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BISTATIC RADAR: MAXIMUM RANGE AND EFFECTIVE AREA

V. I. Kostylev, I. V. Stukalova

Voronezh State University

A monostatic radar refers to a radar system which has the transmitter and receiver located at the same site. Maximum range and effective area are important characteristics of monostatic radar. Another types of radar is bistatic radar; a bistatic radar configuration employs a single transmitter and a single receiver that are not collocated. In this paper we study maximum range and effective area of bistatic radar.

Bistatic radar target observation, i.e. their detection, measurement of their parameters etc, is possible if the signal power, P_R, of the target at the input of receiver is not less than a specific threshold level $P_{R\min}$ [1, 2]. The value of $P_{R\min}$, characterizing the sensitivity of radar receiver, depends on a number of aspects. These include: the type and intensity of interferences at the input of receiver; the required efficiency of target detection and the accuracy in measuring their parameters; the duration of processing the received signal, as well as a number of other factors [2].

The space region where the condition of $P_R \geq P_{R\min}$ is executed is called *the effective area of bistatic radar*. The effective area of bistatic radar is quite often determined by characteristics of attainable accuracy of measurements [1] or by target detection characteristics [3, 4]. In this case, a certain level of signal-to-noise ratio is specified. This is necessary for target detection with a required probability of correct detection at a given probability of false alarms; or for measurement of target parameters with a specified error variance of measurement. In this case, the space region is assumed to be an effective area of bistatic radar if a target with a specified RCS remains in this area and the signal-to-noise ratio at the receiver is not less than a certain level [4].

The form and size of the effective area of bistatic radar depends both on receiver sensitivity (i.e. on $P_{R\min}$), as well as on other aspects such as: the power of the emitted signal; RCS of target; the directional diagram of the receiving and transmitting antennas; atmospheric attenuation of electromagnetic waves; reflection of electromagnetic waves from the ground surface, as well as other factors [1, 2, 5].

In order to find the main criteria determining the effective area of bistatic radar, let us consider an elementary case, namely, let us assume that the target is situated in a uniform free space where attenuation of electromagnetic waves is absent.

Let us imagine that at some point within the space located at distance, R_1, from the transmitter there is a target. Electromagnetic waves emitted by the transmitter have a spherical wavefront, limited by the directional diagram of the transmitting antenna at a sufficiently large distance from the transmitter. Therefore, the power flux density of the direct electromagnetic wave in the vicinity of a target is:

$$\Pi_\Gamma = \frac{P_T g_T}{4\pi R_1^2}, \tag{1}$$

where P_T is an output power and g_T is a directivity factor of the transmitting antenna.

It is known [2, 5] that the directivity factor, g, of any antenna and its gain coefficient, G, are related by the simple formula:

$$G = \eta_A g, \tag{2}$$

where η_A is the efficiency factor of the antenna. As a rule, the efficiency factor for radar antennas is rather high and therefore one can assume that $G = g$.

Thus, instead of (1) one can write down:

$$\Pi_\Gamma = \frac{P_T G_T}{4\pi R_1^2}, \tag{3}$$

where G_T is the gain coefficient for the transmitting antenna. In the vicinity of the phase centre for the receiving antenna of bistatic radar, the power flux density of the wave reradiated by the target is expressed as:

$$\Pi_R = \frac{P_T G_T \sigma_b}{(4\pi)^2 R_1^2 R_2^2}, \tag{4}$$

where just as previously, σ_b is the bistatic RCS of the target, and R_2 is the range of the target relative to the receiver. Multiplying the value of Π_R by the effective area, A_R, of the receiving antenna, one can find the power of the reflected signal incoming to the input of the matched receiver:

$$P_R = \frac{P_T G_T A_R \sigma_b}{(4\pi)^2 R_1^2 R_2^2}. \quad (5)$$

From antenna theory it is known that there is a correlation between the effective area of antenna, A, and its power gain factor G:

$$A = \frac{G\lambda^2}{4\pi}, \quad (6)$$

where λ is a wavelength. With the account of (6) the power of received signal is:

$$P_R = \frac{P_T G_T G_R \lambda^2 \sigma_b}{(4\pi)^3 R_1^2 R_2^2}. \quad (7)$$

where G_R is the gain coefficient of the receiving antenna.

Formula (7) is known as the equation of bistatic radar. Obviously, assuming in (7) $\sigma_b = \sigma_m$ and $R_1 = R_2 = R$, one can obtain the widely known (see, for example, [2]) equation of monostatic radar from the equation of bistatic radar

$$P_R = \frac{P_T G_T G_R \lambda^2 \sigma_m}{(4\pi)^3 R^4}. \quad (8)$$

Formula (8) presumes that, in general case of monostatic radars both the transmitting and receiving antennas can be different. If a pulse monostatic radar receiver and transmitter can be connected to one antenna with the help of an antenna switch then, $G_T = G_R = G$, and monostatic radar's equation can be further simplified as:

$$P_R = \frac{P_T G^2 \lambda^2 \sigma_m}{(4\pi)^3 R^4}. \quad (9)$$

Let us note once again that the Equation of bistatic radio-location (7) was obtained in the assumption of the absence of losses. In the scientific literature regarding bistatic radars it is commonly accepted to account for any losses using the following coefficients [6, 7]: L_{P1} = losses which appear during wave propagation in the area of the transmitter-target; L_{P2} = losses which appear during wave propagation in the area of the target-receiver; L_s = system losses. Thus, the equation of bistatic radio-location can be written in the form [7]:

$$P_R = \frac{P_T G_T G_R \lambda^2 \sigma_b}{(4\pi)^3 R_1^2 R_2^2 L_{P1} L_{P2} L_s}, \quad (10)$$

which is transformed into the monostatic case as:

$$P_R = \frac{P_T G^2 \lambda^2 \sigma_m}{(4\pi)^3 R^4 L_P L_s}. \quad (11)$$

Thus, Equations (7) and (10) generalize the equations of monostatic radio-location.

Let us introduce notation

$$\Omega = \frac{P_T G_T G_R \lambda^2}{(4\pi)^3}, \quad (12)$$

which allows us to rewrite the Equation of bistatic radar (7) in the form of:

$$P_R = \frac{\Omega \sigma_b}{R_1^2 R_2^2}. \quad (13)$$

The expediency of introducing parameter Ω is due to its independence from the target coordinates. However, the bistatic RCS of a target, σ_b, considerably depends on its location, even in the simplest case when the target is an absolutely conductive sphere with an ideal shape (see, for example, [8]).

Nevertheless, in the scientific literature [3, 6, 7, 4] regarding bistatic radars, the effective area is usually determined with the assumption that $\sigma_b = \sigma_0$ = const. Following this tradition, let us obtain the dependency of received signal power, P_R, from the target location. To perform this procedure let us locate coordinate origin, O, in the middle point of the baseline between the receiver and transmitter. In this case, the distance, ρ_0, of the target relative to the coordinate origin is equal to the median length of the bistatic angle. OX axis is directed in such a way that it passes through the phase centres of the transmitter and receiver antennas; while OY axis is chosen to be arranged in the bistatic plane perpendicular to OX axis. To continue the generalisation, we introduce the normalized coordinates of the target, $\hat{x} = x/L$ and $\hat{y} = y/L$, where L is the base of bistatic radar. Thus, the normalized coordinates of the transmitter are, $\hat{x}_T = -0.5, \hat{y}_T = 0$, while those of a receiver are, $\hat{x}_R = 0.5, \hat{y}_R = 0$. Fig. 1 demonstrates the depen-

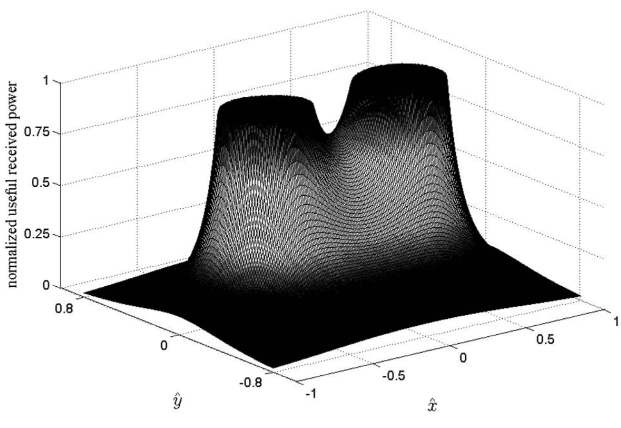

Fig. 1

dence of normalized useful received power on the target location.

Note that the formula (13) was deduced for the case of spherical waves. The wave is not spherical in the near-field area of the transmitting antenna, therefore, if a target is located in this area it is impossible to apply formulae (7) and (13). Similarly, if a target is located quite close to the phase centre of the receiving antenna, then a secondary wave re-emitted by the target is also a non-spherical one, hence, formulae (7) and (13) again become incorrect.

As it follows from Fig. 1, the possibility of operating with the required characteristics in the case of bistatic radar depends not only on the target range, but also on the angle position (azimuth) of a target relative to the base line of the radar. Therefore, the use of the idea of *the radar range* [4] in the bistatic case is not as fruitful as in the monostatic scenario.

Substituting in (7) $P_R = P_{R\min}$, where just as earlier $P_{R\min}$ was the sensitivity of the receiving system, (the minimum possible value of received power) we obtain the relation:

$$(4\pi)^3 R_1^2 R_2^2 P_{R\min} = P_T G_T G_R \lambda^2 \sigma_b, \qquad (14)$$

describing the closed line arranged in bistatic plane. This closed line is the external boundary of the effective area of bistatic radar. Assuming independence of bistatic RCS, σ_b, on parameters of the bistatic triangle, the external boundary of the effective area of bistatic radar is a locus of points where a product of distances to two specified points is a constant value. In our case, these are the points where the transmitter and receiver are situated. In mathematics these curves are known as Cassini curves.

Sometimes the ratio of power of the probing signal to receiver sensitivity, $\varsigma = P_T / P_{R\min}$, is chosen as independent radar characteristics and it is called *energy potential* [5]. By using the idea of energy potential, the formula describing the external boundary of an effective area of bistatic radar takes the form:

$$(4\pi)^3 R_1^2 R_2^2 = \varsigma G_T G_R \lambda^2 \sigma_b.. \qquad (15)$$

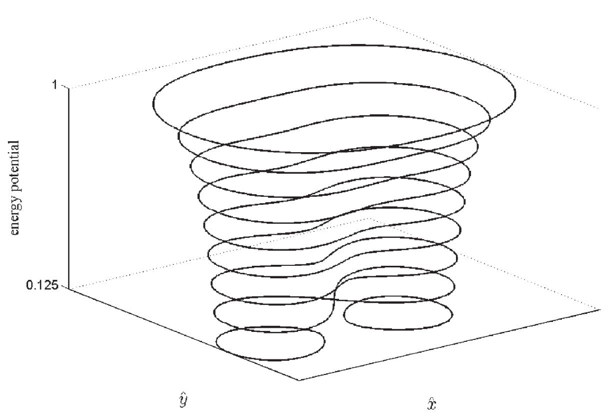

Fig. 2

In Fig. 2, external boundaries of the effective area of bistatic radar are presented for different values of the energy potential.

For analysis of effective areas of bistatic radar it seems useful to introduce the geometric mean value;

$$\rho = \sqrt{(R_1 R_2)_{\max}} = \sqrt[4]{\frac{\varsigma G_T G_R \lambda^2 \sigma_b}{(4\pi)^3}}, \qquad (16)$$

for the range of a target situated on the Cassini oval. Parameter ρ is then called *effective range of bistatic radar*.

Through analysis (16), it is possible to make the following conclusions:

1. The effective range of bistatic radar quite slowly increases with the increase of energy potential. For example, in order to raise the effective area of bistatic radar by 2 times, one should increase its energy potential by 16 times, i.e. by 12 dB.

2. The effective range of bistatic radar also slightly depends on the gain coefficients of the transmitting and receiving antennas: in order to increase the effective area for bistatic radar twice, it is necessary to increase both of these coefficients by 6 dB.

3. Dependence of the effective area of bistatic radar on the wave length is more complicated than it seems from (16), since parameters G_T, G_R and σ_b considerably depend on the wave length.

4. Dependence of the effective area of bistatic radar on RCS is also rather weak. Increase or decrease of RCS by several decibels is equal, by implication, to the correspondent change of the energy potential by the same value in decibels. Therefore, in order to increase (decrease) the effective area of bistatic radar by 2 times, RCS should be increased (reduced) by 12 dB.

As it is seen from Fig. 2, where a high energy potential is present, the shape of the external boundary for the effective area of bistatic radar is close to an elliptic one. One can show [3], that the effective area of bistatic radar has a similar convex form under condition $\rho \geq L/\sqrt{2}$, where L is the bistatic base of the radar. With a decrease in the energy potential, the effective area of bistatic radar is extended: for $L/2 < \rho < L/\sqrt{2}$, this area of bistatic radar remains a simple connected domain, however, it ceases to be convex. For $\rho = L/2$, the boundary of the effective area of bistatic radar is a closed curve known as Bernoulli lemniscate. This is the case when the effective area of bistatic radar consists of two symmetrical regions with the same shape and size contacting each other in one and the same point, the middle point of the base line. Under further decrease of the energy potential (for $\rho < L/2$), the effective area of bistatic radar ceases to be simply connected; it is divided into two separate areas surrounding the transmitter and receiver. In this case, the external boundary of the effective range of the radar is a so-called [3] degenerated Cassini oval. More detailed discussion of the properties of effective areas of bistatic radar bounded with Cassini curves is given in [3].

Let us be reminded that for monostatic radar, the effective region is a circle and its area is equal to:

$$S = \pi R_{\max}^2, \tag{17}$$

where $R_{\max}$ is the maximum range of monostatic radar.

The area of the effective region for bistatic radar can be calculated by the formula:

$$S = 4\int_0^{\pi/2}\int_0^{\rho_0} r dr d\varphi, , \tag{18}$$

where ρ_0 is a target range relative to the coordinate origin point, O, situated in the middle point of the base line of radar (ρ_0 is equal to the length of the median in the bistatic triangle) ; r and φ are polar coordinates of the target relative to the same coordinate origin O. The value of parameter, ρ_0, can be calculated by the formula [3]:

$$\rho_0^2 = \frac{1}{4}L^2\cos 2\varphi \pm \sqrt{\rho^4 - \frac{1}{16}L^4\sin^2\varphi}. \tag{19}$$

Formula (19) represents some other settings of the boundary for the effective area of bistatic radar.

Averyanov [3] has obtained that for $\rho >> L/2$, the area of the effective region of bistatic radar can be calculated by the approximate formula*:

$$S \simeq \pi\rho^2\left(1 - \frac{1}{64}\frac{L^4}{\rho^4}\right). \tag{20}$$

For the case of $\rho = L/2$, the area of the effective region of bistatic radar can be calculated exactly by the formula [3]

$$S = \rho^2 . \tag{21}$$

In the case of $\rho << L/2$, the area of the effective region for bistatic radar is [3]

$$S \simeq \pi\frac{\rho^4}{L^2}. \tag{22}$$

As one can see from the formulae (20)—(22), the influence on radar parameters such as energy potential, gain coefficients of transmitting and receiving antennas, wave length, as well as the target RCS on the value of the area for the effective region of bistatic radar is determined by the effect of the indicated parameters on the effective range length of bistatic radar.

From a comparison of (17) to (22), it follows that for equal radar parameters and target RCS the effective range of bistatic radar is less in area than the effective region of monostatic (equivalent) radar [3].

One of the assumptions made in this article was that the derivation of the equation of bistatic radar did not take into consideration the Earth's influence on the process of radio-wave propagation. This assumption allows us to consider both electromagnetic wave spherical forms; one emitted by the transmitting antenna and the other re-emitted by the target. If the bistatic plane is near the ground surface then the formulae represented in this article should be corrected. For example, instead of (13), one should write:

$$P_{\mathrm{R}} = \frac{\Omega\sigma_b}{R_1^\alpha R_2^\alpha}, \tag{23}$$

where α is a real parameter that does not exceed 2. Certain values of this parameter depend on a number of factors: a degree of smoothness or roughness of the ground surface; electrophysical properties of the ground; the height of transmitter; receiver and target over the ground surface; polarization of the probing and received electromagnetic waves, and so on. By simple participation of one of the authors in experimental study (see, for example [9], [10]), it was determined that in a certain situation parameter, α, can be close to 1.5.

According to (23) the main equation of bistatic radar must be also changed:

$$R_1^\alpha R_2^\alpha = \varsigma G_T G_R \lambda^2 \sigma_b /(4\pi)^3 . \tag{24}$$

However, from (24) one can simply obtain:

* According to [3] for $\rho > L$ an error of the formula does not exceed 10 %.

$$R_1^2 R_2^2 = \left[\frac{\varsigma G_T G_R \lambda^2 \sigma_b}{(4\pi)^3} \right]^{2/\alpha}, \qquad (25)$$

and from here it directly follows that just as before, the external boundary of the effective area of bistatic radar has the shape of a Cassini oval. Thus, taking into account the Earth's influence results only in a scaling change for Cassini curves.

As for the effective range of bistatic radar, then in consideration of the Earth's influence it is:

$$\rho = \left[\frac{\varsigma G_T G_R \lambda^2 \sigma_b}{(4\pi)^3} \right]^{\frac{1}{2\alpha}}. \qquad (26)$$

In particular, for $\alpha = 1.5$, expression (26) takes the form of:

$$\rho = \frac{\sqrt[3]{\varsigma G_T G_R \lambda^2 \sigma_b}}{4\pi}. \qquad (27)$$

Thus, if the radar bistatic plane is arranged near the ground surface, then in order to increase the effective range of radar by 2 times it is sufficient to increase the energy potential of the radar by only 9 dB, not by 12 dB as was previously noted in the article above.

Until now, we have assumed that the antennas of the transmitter and receiver are omnidirectional and that the gain coefficients of transmitting, G_T, and receiving, G_R, antennas are constant (independent from target direction). The use of phased array as the receiving and transmitting antennas of bistatic radar can require taking into account the dependence of parameters G_T and G_R on target location while deriving the equation of bistatic radar. A detailed discussion of this issue can be found in [4].

REFERENCES

1. *Shirman Ya.D.* (ed.) (1970) Teoreticheskiye osnovy radiolokatsii (Radar Fundamentals) in Russian. Russia: Sovietskoye Radio.

2. *Dulevich V.E.* (ed.) (1978) Teoreticheskiye osnovy radiolokatsii (Radar Fundamentals) in Russian. Russia: Sovietskoye Radio.

3. *Aver'yanov V.Ya.* (1978) Raznesennye radiolokatsionnye stantsiy i sistemy (Radar Stations and Systems with Space Diversity) in Russian. Belarus: Nauka i Tekhnika.

4. *Chernyak V.S.* (1998) Fundamentals of Multisite Radar Systems. Multistatic Radar and Multiradar Systems. Amsterdam: Gordon and Breach Science Publishers.

5. *Finkel'shteyn M.I.* (1983) Osnovy radiolokatsii (Fundamentals of Radar) in Russian. Russia: Radio i Svyaz.

6. *Willis N.J.* (1990) Bistatic Radar, chap. 25 in Skolnik M.I. (ed.): "Radar Handbook". New York: McGraw-Hill Book Company.

7. *Dunsmore M.R.B.* (1993) Bistatic Radars; Chapter 11 in "Advanced Radar Techniques and Systems" (G. Galati ed.). Peter Peregrinus.

8. *Nathanson F.E., Reilly J.P., Cohen M.N.* (1999) Radar Design Principles: Signal Processing and the Environment (2nd Edition). New York: SciTech Publishing.

9. *Cherniakov M., Salous M., Jancovic P., Abdullah R., Kostylev V.* (2005) Forward Scattering Radar for Ground Targets Detection and Recognition, Proc. 2nd Annual Technical DTC Conference. Edinburgh, U.K.

10. *Cherniakov M., Salous M., Kostylev V., Abdullah R.* (2005) Analysis of Forward Scattering Radar for Ground Target Detection, Proc. 2nd European Radar Conference, France. Paris. P. 145—148.