NONLINEAR SCATTERING OF LASER PULSE BY ELECTRON IN PERIODIC POTENTIAL

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The classical model of a nonlinear scattering of intense pulse of laser radiation by an electron, taking place in a periodic potential field, is discussed. On the basis of the approximation of Kapitza-Dirac the complete analytical solution in the case of compressed pulse is obtained. The form of scattered radiation and its spectral distribution for various forms of initial laser pulses are found.

1. INTRODUCTION

The process of the high harmonic generation as the result of the action of intense laser radiation on various systems (atomic and molecular gases, metal films, glass structures, carbon nano tubes and others) is a subject of this study. The progress in this area has given, in particular, sources of a coherent ultra-violet radiation and cutting of duration of a scattered pulse up to atto second. The radiation with such unique properties has practical application at the examination of biological objects and study of the rate of the fast chemical reactions.

Qualitatively, the mechanism of harmonic generation on atomic gases for a low-frequency field is explained on the basis of the semiclassical Corkum's model (Corkum 1993). According to this model the electron generates harmonics during the scattering on the potential of the atomic core after nonlinear ionization. The motion of an electron, in this model, is considered as propagation of a wave packet in a field of an electromagnetic wave (Rae and Barnett 1993). The difficulties of complete quantum consideration of the high harmonic generation in strong laser fields are not completely overcome till now, though the satisfactory consideration of the process is obtained both for atomic gases, and for gases of diatomic molecules (Chin and Golovinski 1995, Chin et al 1995).

The harmonic generation in periodic structures (crystals) was considered theoretically in (Kalman and Brabec 1995, 1996). In these papers the field of strong laser radiation was taken as classic, and the remaining part of the problem was solved as quantum. The high harmonic generation in such model arises as a result of a spontaneous radiation of electrons scattered on ions of a lattice, at a motion under the action of a strong laser field. Thus, however, the plasma effects, essential in available experiments were not taken into account, and this impedes comparison of results of the theoretical investigations with the experimental data.

Recently serious successes are reached in preparation and investigation of carbon nanotubes, containing one or several layers of cylindrical structures by a diameter from one up to several tens nm and length up to a several microns (Eletzkiy 2002). They have different structure, a different period of a lattice and have various types of conductance (Mintmire J. et al 1992, Ebbesen T. W. et al 1996). Nanotubes contain only few layers of carbon that excludes the nonlinear effects in near wall plasma under the action of laser radiation. At the same time the translation symmetry in them is precisely exhibited. The development of physics and technique of the nanostructures boosts theoretical investigations of interaction of laser radiation with periodic microsystems. We shall consider the model of a nonlinear scattering of a laser pulse of high intensity on such system stipulated by nonlinear motion of an electron in the field of periodic structure and a laser wave.

The features of a physical picture in many respects are connected with the shape of a laser pulse. In the case of asymmetrical pulses with the effective duration of a pulse T, the average value of the electric strength $\bar{F}$, is defined by

$$\bar{F} = \frac{1}{T}\int_0^T F(t)dt,$$

and it can be comparable with the peak value $F_{\max}$. The electron under the action of the pulse gains a kinetic energy comparable with the energy of its interaction with a solid-state comb. The basic part of radiation of such electron generates after interaction with a field and is determined by the radiative friction at a motion in periodic potential of a lattice. This process is nearly the same that is well known radiation in channels.

On the other hand, when $\bar{F} << F_{\max}$, the energy of an electron after interaction with a separate laser pulse is approximately equal zero, and the value of radiation after the action of the pulse can be neglected. The radiation in this case happens only during interaction of an electron with a laser pulse. The mechanism of this process is not enough investigated and it is the subject of the this paper.

2. GENERAL EQUATIONS

That fact, that a spectrum of energies of an electron in combined field of an intense electromagnetic wave and solid-state comb is continuous, allows to take advantage of classical approach in description of a motion of an electron. Besides we shall assume that periodic structure has size L, that is much less than a wavelength of laser radiation λ, that will allow us to be restricted by dipole approximation. We take one-dimensional model of periodic potential of a lattice how it is made in the Frenkel theory of a motion of dislocations (Frenkel 1950):

$$U = \frac{f_0}{k}\left[1 - \cos(kx)\right], \tag{1}$$

f_0 is the field amplitude of a solid-state «comb» and parameter k is defined by a spatial period of a lattice a as follows: $k = 2\pi / a$. The force f, acting on an electron in such periodic structure, has a form:

$$f = -\frac{\partial U}{\partial x} = -f_0 \sin(kx). \tag{2}$$

Let's choose the linear polarization of the laser field along an axis of nanotube x, provided that the motion of an electron is described by the one-dimensional equation of Newton

$$\ddot{x} = F(t) - f_0 \sin(kx). \tag{3}$$

$F(t)$ is the force of the interaction of an electron with a laser field, depending on time, (we use the atomic system of units, where $m_e = e = 1$). An initial time $t = 0$ we choose so, that $F(0) = 0$, and the electron is in a minimum of the potential energy of the lattice in the point $x = 0$.

If the external field is great ($F_{\max} > f_0$) it is convenient to take advantage of the modified Kapitza—Dirac approximation (Landau and Lifshitz 1973), and the solution of the Eq. (3) can be presented as the sum of two terms

$$x = d + \xi. \tag{4}$$

$\xi(t)$ is the solution representing fast oscillations in a strong laser field, and $d(t)$ takes into account the nonlinear corrections to the motion as a result of interaction with a lattice. Then instead of Eq.(3) we shall obtain two equations

$$\ddot{\xi} = F(t), \tag{5}$$

$$\ddot{d} = -f_0 \sin(k\xi(t)). \tag{6}$$

The initial conditions are $\xi(0) = 0, d(0) = 0$ and $\dot{\xi}(0) = 0, \dot{d}(0) = 0$. The solution of the Eq. (5) for any pulse is expressed in quadratures

$$\xi(t) = \int_0^t \left(\int_0^\tau F(\tau_1) d\tau_1 \right) d\tau = \int_0^t F(\tau)(t - \tau) d\tau. \tag{7}$$

In Eq. (7) we used the formula of Cauchy for iterated integral. After substitution of the solution of Eq. (7) into the Eq. (6), we shall obtain

$$\ddot{d} = -f_0 \sin\left(k\int_0^t F(\tau)(t - \tau) d\tau \right). \tag{8}$$

This allows us at once to calculate the shape of a scattered pulse for various forms and amplitudes of an initial pulse. Really, in a long-field radiation zone of the dipole ($r \gg \lambda \gg L$) the formula (Landau and Lifshitz 1973) for an electric field of the scattered wave is valid:

$$\mathbf{E}(t) = \frac{\ddot{d}(t)\mathbf{e}}{c^2 r} = -\frac{f_0 \mathbf{e}}{c^2 r} \sin\left(k\int_0^t F(\tau)(t - \tau) d\tau \right). \tag{9}$$

Here the vector $\mathbf{e}$ is connected to the unit vector of polarization of laser radiation $\mathbf{e}_0$ and the unit vector $\mathbf{n}$, directed from an electron to a point of observation, as follows: $[[\mathbf{e}_o, \mathbf{n}], \mathbf{n}]$.

3. CALCULATIONS

In particular, for a harmonic laser radiation $F = F_0 \sin(\omega t))$ the solution of the Eq. (5) has the form

$$\xi(t) = -\frac{F_0}{\omega^2} s \sin(\omega t). \quad (10)$$

After substitution of Eq. (10) into Eq.(6), we shall have

$$\ddot{d}(t) = f_0 \sin(z \sin(\omega t)), \quad z = k\frac{F_0}{\omega^2}. \quad (11)$$

For small z the dependence of intensity of a scattered wave is harmonic function of time. For $z > \pi$ the dependence of the field intensity of a scattered pulse as a function of time has the local minimums, with positions determined by the formula $\omega t_k = \arcsin(k\pi / z), \ k = 1,2,3\ldots$.

Generating function for Bessel functions (Bateman and Erdely 1966) is

$$\sin(z \sin\theta) = 2\sum_{n=1}^{\infty} J_{2n-1}(z) \sin[(2n-1)\theta]. \quad (12)$$

It allows us to obtain a spectral decomposition of the scattered wave as

$$\mathbf{E}(t) = \frac{2f_0\mathbf{e}}{c^2 r}\sum_{n=1}^{\infty} J_{2n-1}(z) \sin[(2n-1)\omega t]. \quad (13)$$

Thus, in the spectrum of scattered radiation there will be only odd harmonics as a consequence of the symmetry of the lattice concerning a veering of an axis x on opposite direction at the fixed earlier starting conditions. The intensity of radiation of an electron is given by expression

$$I = \frac{2\ddot{d}^2}{3c^2} = \frac{8f_0^2}{3c^2}\left(\sum_{n=1}^{\infty} J_{2n-1}(z)\sin[(2n-1)\omega t]\right)^2. \quad (14)$$

The intensity $\bar{I}$, averaged on the period of a radiated wave, is possible to present as the sum of the intensities of the monochromatic components:

$$\bar{I} = \sum_{n=1}^{\infty} I_n = \frac{4f_0^2}{3c^3}\sum_{n=1}^{\infty} J_{2n-1}^2(z). \quad (15)$$

As the result the dependence of the average value of intensity is approximated by the expression

$$\tilde{I}_n = \frac{4f_0^2}{3c^3}\exp\left[-\frac{8}{3}z + 100\cdot(1-\exp(1.15n))\right]. \quad (16)$$

in the interval $n < 200, \ 0.01 < z < 100$, with a relative accuracy not exceeding 0.05. The result of calculation of average intensity of radiation of harmonics is given for various values of parameter z in Fig.1. For small z the dependence of intensity of harmonics as a function of its number is smooth. With the growth of the value of the parameter z, the number of local extremes in dependence is grows.

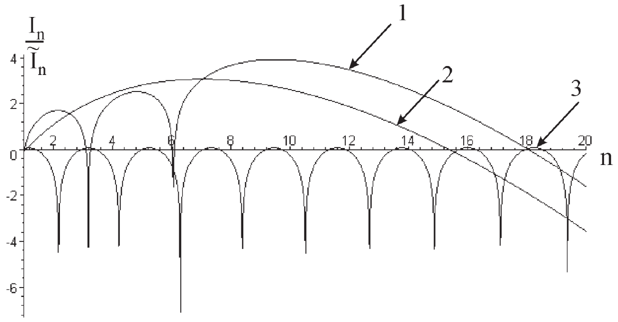

Fig. 1. The dependence of the ratio of average intensity of a radiated harmonic I_n to the approximation by Eq. (16) as a function of the number of the harmonic n. Gauge on an axis of ordinates is logarithmic. The curve 1 corresponds to the value of dimensionless parameter $z \le 1$, and the curves 2 and 3 — $z = 10$ and 100 respectively.

In theoretical investigation of the processes of interaction of the matter with a laser field the radiation usually is considered like a harmonic with the slowly varying amplitude as a function of space coordinates and time (Shore and Kulander 1989). The progress in generation of ultra short pulses has revealed frames of such approximation. Therefore for theoretical description of the processes of nonlinear interaction of the matter with such radiation the various models of a laser field reflecting experimental methods of generation of ultra short pulses were offered. One of the procedures of generation of ultra short pulses is based on the use of the broadband generating mediums, as it is done in titanium-sapphire lasers (Akhmanov et al 1988), for which the radiation is represented by the sum of harmonics of frequency Ω with close amplitudes modulated by high frequency $\omega >> \Omega$

$$F(t) = \cos(\omega t - \varphi)\sum_{l=0}^{N} F_l \sin(l\Omega t). \quad (17)$$

If the amplitudes of harmonics are identical ($F_l = F_0$) the sum in Eq. (17) is easily calculated as

$$F(t) = F_0 \frac{\sin\left(\frac{1}{2}N\Omega t\right)}{\sin\left(\frac{1}{2}\Omega t\right)}\cos(\omega t - \varphi). \quad (18)$$

Such radiation gives an infinite sequence of pulses with frequency of repetition Ω. The width of a separate pulse linearly decreases with the growth of the number of harmonics N in the sum. Thus for formation of ultra short pulses, with the width comparable with a period of high-frequency oscillation, it is necessary to combine rather big ($N \cong 10^2$) number of harmonics. By a special choice of amplitudes of terms of the sum in Eq.(17) as σ-factors of Fejer (Lantzosh 1961)

$$\sigma_l = \frac{1}{N}\left[1 + (-1)^{l-1}\right]\sin\left(\frac{l\pi}{N+1}\right), \quad (19)$$

it is possible, practically with no changing of a pulse shape, considerably (up to $N \sim 10$) reduce the number of terms of the harmonic decomposition. In this case the amplitude of a laser field is represented as

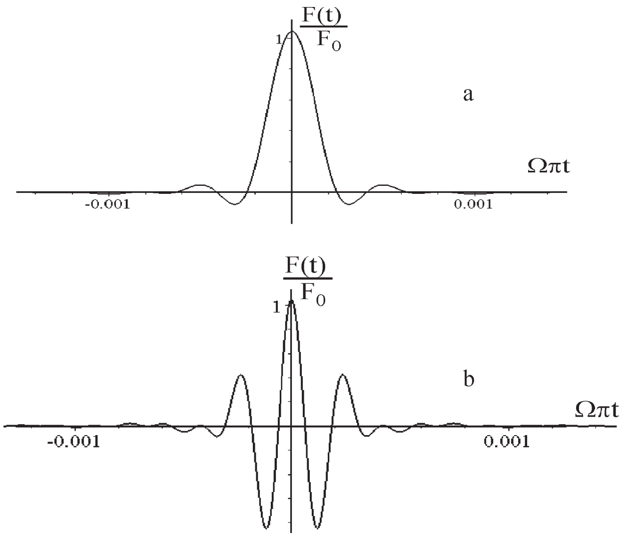

Fig. 2. The shapes of an incoming ultra short pulses of the form of the Eq. (20) at number of harmonics $N = 40$, pulse-repetition frequency $\Omega = 0.01$. Modulating frequency ω is equal 0.2 and 0.8 accordingly

$$\mathbf{F}(t) = C\mathbf{F}_0 \cos(\omega t)\sum_{l=1}^{2m-1} \sigma_l \cos(l\Omega t). \quad (20)$$

The normalization coefficient is defined by the expression $1/C = \sum_{l=1}^{2m-1}\sigma_l$ and $m = \frac{N+1}{2}$.

The shapes of the pulses for various values of the ratio of frequencies $x = \Omega/\omega$ are given in a Fig. 2. It is evident, that at the diminution of the parameter x the pulse becomes more symmetric. The solution of the Eq. (5) in a field of a wave given by the Eq. (20) has an analytical form.

Time dependence of displacement of an electron $\xi(t)$ for pulses of the various shapes is presented in a Fig. 3. The curve 3 corresponds to the asymmetrical pulse, with the shape figured in the Fig. 2a. In this case, the electron velocity is increased as the result of interaction with each laser pulse. The curve 1 corresponds to a symmetric pulse, which shape is figured in a Fig. 2b. After interaction with a pulse an electron remains in the rest, and the radiation happens during interaction with a laser pulse. The curve 2 corresponds to the laser pulse of the intermediate shape.

Substituting the solution of the Eq. (23) into the Eq.(6), and taking into account connection of potential functions with functions of multiple arguments, it is easy to obtain the harmonic decomposition of a scattered pulse. The formulas, however, are rather complex and are not presented here.

The number of analytical evaluations necessary for definition of the weights of harmonics grows with magnification of intensity of the laser field. Therefore in strong fields more convenient is to use the algorithm based on trigonometric interpolation of the equidistant data as in (Lantzosh, 1961). Besides,

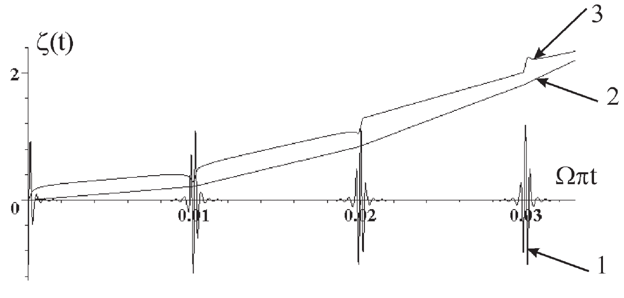

Fig. 3. Time dependence of the displacement of electron

this algorithm is applicable when the harmonic representation of a laser pulse is not known or poorly converges. Coefficients of the Fourier decomposition of the electric field strength of a scattered wave

$$E(t) = \sum_{l=-N}^{N} c_l \exp(ilt) \qquad (25)$$

are expressed with the use of the values of function $E(x_j)$ in equidistant points of any intervals of periodicity $[t_0, t_1]$ by the formula

$$c_l = \frac{1}{2N} \sum_{j=-N}^{N} E(x_j) \exp(-ilx_j). \qquad (26)$$

Here $x_j = \frac{\pi}{2}\left[\frac{j}{N}(t_1 - t_0) + t_0\right]$. The calculations according to the Eq. (26) do not depend on the form of the laser pulse, and depend only on the number of the harmonics *N* taken into account. The result of the calculation of the spectral distribution of the scattered pulse is given in the Fig. 4.

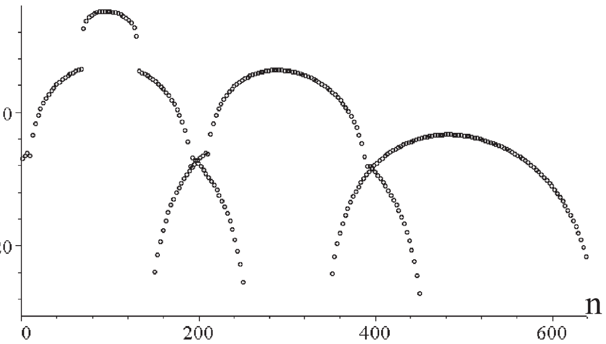

Fig. 4. Spectral distribution of scattered radiation for the pulse of the form presented in the Fig. 2.b. On an abscissa axis we plotted the number of a harmonics, and on an axis of ordinates the intensity averaged over the period, and normalized on the unit in the maximum of radiation. The gauge on the axis of ordinates is logarithmic

In addition to the summation of harmonics there are also other experimental procedures permitting to shape limiting short (down to practically unipolar) pulses (Jones et al 1983). Adequate complete basis for decomposition of such pulse can be taken as wavelets or frames (Novikov and Stechkin 1998). For the frames the basis can be constructed, for example, as the derivatives of Gauss function

$$\psi_m = \frac{d^m}{dt^m} \exp\left(-t^2\right), \quad m > 1. \qquad (27)$$

The double integration of the function $\xi(t)$ on time again will give decomposition, but in the other basis with $m' = m - 2$. For $m = 2$ the result of integration is Gaussian: $\xi(t) = \exp(-t^2)$. Earlier we used the basis with $m = 2$:

$$\psi_2(t) = 2(2\alpha^2 t^2 - 1)\exp(-\alpha^2 t^2), \qquad (28)$$

and it allowed us to describe a diffraction and focusing of a ultra short pulse (Mikhailov and Golovinski 2000).

For a pulse of an incident wave represented by the one frame from the basis of Eq. (28), the amplitude of the scattered wave has the form

$$\mathbf{E}(t) = \frac{\mathbf{e}}{c^2 r} \sin\left[k \frac{F_0}{\alpha^2} \exp\left(-\alpha^2 t^2\right)\right]. \qquad (29)$$

The shape of the laser pulse given by Eq. (28) is figured in the Fig. 5a. Characteristic duration of this pulse is equal $1/\alpha$. The dependence of electric field strength of a

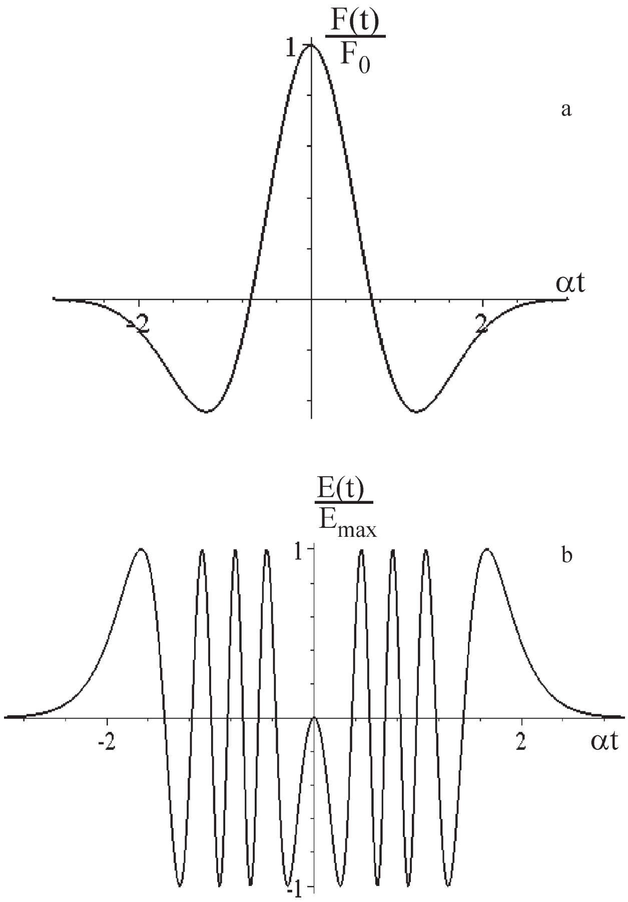

Fig. 5. a — The shape of the laser pulse for symmetric basic frame when the value of dimensionless parameter $k\frac{F_0}{\alpha^2} = 8\pi$. b — The form of the scattered pulse

scattered pulse as a function of time is plotted in the Fig. 5b. Decomposing $E(t)$ in the Fourier integral, we shall have

$$\mathbf{E}(\omega') = \frac{\mathbf{e}\beta_0}{c^2 r\omega}\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!\sqrt{2k+1}}\times$$
$$\times\exp\left(-\beta_k^2\right)\cdot\left(k\frac{F_0}{\alpha^2}\right)^{2k+1}, \qquad (30)$$

where

$$\beta_k = \frac{\beta_0}{\sqrt{2k+1}}, \quad \beta_0 = \frac{\omega'}{2\alpha}. \qquad (31)$$

According to the Eq. (30) the product $E(\omega')\omega'$ depends only on dimensionless parameter β_0. The dependence of the Fourier spectrum of the scattered pulse as a function of intensity of the laser radiation is nonmonotone.

4. CONCLUSION

The model of motions of an electron in the field of electromagnetic wave and periodic spatial structure is described by the classical equations, and it is capable explain a conversion of an initial pulse and, in particular, high harmonic generation. If the kinetic energy of an electron in the laser wave considerably exceeds the potential energy, then oscillations, defined by the interaction with electromagnetic wave can be described separately. The solution of this part of a problem is expressed in quadratures. For many of typical pulses the integration of the equation of motion of an electron in elementary functions is possible. In particular, it is valid for harmonic field and for ultra short pulses represented by the Fourier decomposition and by the frames, which are derivative of the Gaussian functions.

The solution of the problem of defining the shape of the scattered wave is linked algebraically to the solution of the equation of motion. Thus a physical picture of the process is clear for various pulses close to symmetric and pulses possessing considerable asymmetry. In this model it is possible to obtain analytical expressions featuring conversion of arbitrary laser pulse.

The spectral distribution of a field of a scattered wave is determined by the Fourier transform of a field of a scattered wave for solitary pulses of laser radiation and by the coefficients of the Fourier decomposition for periodic laser pulses. The Fourier transform of the scattered pulse for laser radiation of arbitrary Gaussian frame can be calculated as the power series on intensity with coefficients depending on dimensionless parameter. For the harmonic laser signal the Fourier coefficients are expressed through Bessel functions. The results of calculations testify for field of a sine-type wave, that intensity of harmonics decreases with the growth of parameter z according to the power law. With increasing of the number of a harmonics n the rate of their slope exceeds exponential. The approximating formula ensuring in a wide interval of the changes z and n the relative error, not exceeding 5%, is obtained for the intensity of harmonics.

For ultra short pulses in many practically interesting cases the spectral distribution of scattered radiation can be represented by several first terms of decomposition of amplitude of a harmonics in powers series on amplitude of electromagnetic wave. The development of the model, offered in the present paper, can be derived in several directions. Important at build-up of the more detailed theory is to take into account the features of the carbon nanotubes structure such as positions of carbon atoms in the vertexes of hexagons. The sizes of actual nanotubes achieve several microns, and the interest has an outcome from the boundaries of the dipole approximation and the study of the problem of the coherence of radiation scattered from different electrons of the structure. On the other hand, the application of the given model to the processes of a scattering of a signal on atomic and molecular gases is possible too.

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